Non-renewable Resources: Optimal Extraction
Categories of Natural Resources

• Nonrenewable vs. Renewable

  – Nonrenewable – finite quantity, rate of generation insignificant compared with rate of use.

  – Renewable – high rate of generation or regeneration.

  – With nonrenewables we are concerned with determining efficient inter-temporal consumption.
Defining Dynamic Efficiency

• When a policy or program produces streams of benefits and costs over time, it is *dynamic*, rather than *static*.

• In a dynamic setting, the economically efficient allocation maximizes the present value of net benefits.

• At this allocation, \( PV(\text{marginal net benefits}) \) are equal across time periods.

• If this weren’t true, it would be possible to increase the present value of net benefits by re-allocating consumption across time periods.
Parameters of Our Two-period Problem: Extraction of 20 Barrels of Oil.

Demand: $MB = 8 - 0.4 \times q$

Supply: $MC = \$2/\text{unit}$

Stock of resource = 20 units

Discount rate: $r = 0.10$

PVNB ≡ present value of net benefits
Problem with Static Efficiency and Non-renewables

Demand for oil: \( MB = 8 - 0.4q \)
Problem with Static Efficiency and Non-renewables

Demand for oil: $MB = 8 - 0.4q$

PERIOD #1

$MB(Q)$

PERIOD #2

$MB(Q)$

15+15=30 > 20 units available
First Candidate for Two-period Consumption Allocation

- Candidate 1: Extract 15 in period 1, and leave whatever is left over (5) for consumption in period 2.

\[ \text{NMB(shaded)} = 26 \]

\[ \text{PVNB} = 26 + \frac{22}{1+.10} = 26 + 20 = 46 \]
Second Candidate for Two-period Consumption Allocation

- Candidate 2: Extract 5 in period 1, and leave 15 for consumption in period 2.

\[
NMB = 22
\]

\[
NMB = 26
\]

\[
PVNB = 22 + \frac{26}{1+.10} = 22 + 23 = 45
\]
Algebraic Solution to Dynamically Efficient Allocation in Two Periods

- In a dynamic setting, the economically efficient allocation maximizes the present value of net benefits. At this allocation, PV(marginal net benefits) are equal across time periods.

\[ PV(MNB)_1 = PV(MNB)_2 \]

\[ PV(MB - MC)_1 = PV(MB - MC)_2 \]

\[ 8 - 0.4 \times q_1 - 2 = \frac{8 - 0.4 \times q_2 - 2}{(1.10)^1} \]

\[ q_1 + q_2 = 20 \quad so \quad q_2 = 20 - q_1 \]

Substituting:

\[ 6 - 0.4 \times q_1 = \frac{6 - 0.4 \times (20 - q_1)}{(1.10)^1} \]

\[ \Rightarrow q_1^* = 10.239, \quad q_2^* = 20 - q_1^* = 9.761 \]

\[ p_1 = 8 - (0.4 \times 10.239) = \$3.90 \]

\[ p_2 = 8 - (0.4 \times 9.761) = \$4.10 \]
Non-renewable Resource Extraction: The Two-period Model

Marginal Net Benefit in Period #1 ($)

PV of \( MB - MC \) in Period #1

PV of \( MB - MC \) in Period #2

Marginal Net Benefit in Period #2 ($) discounted at 10% r

Q in Period #1 → 0

Q in Period #2 ← 0

5.45

0 5 10 15 20

20 15 10 5

0 1 2 3 4 5 6
Dynamically Efficient Allocation in the Two-period Model

From demand function, \[ MB = 8 - 0.4q \]
- \( p_1 = 3.90 \)
- \( p_2 = 4.10 \)

\[ q_1 = 10.239 \]
\[ q_2 = 9.761 \]
Dynamic Efficiency with Constant Marginal Extraction Costs

Period #1

\[ p_1 = 3.90 \]
\[ \text{MEC} \]
\[ \text{demand} \]
\[ q_1 = 10.239 \]

\[ \text{MUC} \text{ is } \textit{marginal user cost} \]
\[ \text{MEC} \text{ is } \textit{marginal extraction cost} \]

Period #2

\[ p_2 = 4.10 \]
\[ \text{MEC} \]
\[ \text{demand} \]
\[ q_2 = 9.761 \]

\[ \text{MUC}_1 = 1.90 \]
\[ \text{MUC}_2 = 2.10 \]
Scarcity and Marginal User Cost

- Marginal user cost (or scarcity rent) of current consumption is the opportunity cost of forgone future consumption.

- For non-renewables, MUC = P - MEC

- This extra cost is a negative externality from the extraction of non-renewable resources.

- Must be internalized for market equilibrium allocation to be efficient.
The Hotelling Rule

- At the dynamically efficient extraction allocation of a non-renewable resource with constant marginal extraction cost, the marginal user cost rises over time at the rate of interest (the opportunity cost of capital).

\[
\frac{\text{MUC}}{\text{MUC}} = r \quad \text{or} \quad \frac{P - \text{MEC}}{P - \text{MEC}} = r
\]

\[
\frac{\partial \text{MUC}}{\partial t} = r
\]

*In the discrete, two-period case:*

\[
\frac{\Delta \text{MUC}}{\text{MUC}} = \frac{\text{MUC}_2 - \text{MUC}_1}{\text{MUC}_1} = r
\]

- Therefore, price also rises at the rate of interest [since MEC is constant – refer fig. on pg. 12]
- No-arbitrage condition: if it were possible to make more ($) by shifting consumption around, the private owner would do that.
Assumptions of Hotelling Model

• Constant marginal extraction costs: only MUC changes over time.

• Private, competitive owners of non-renewable resources: property rights are well defined.

• Future price path is known (or “equilibrium in expectations”)

Generalizing from 2 Periods to N Periods

• Generalizes to the n-period case.

• Hotelling rule still holds for constant MEC.

• Exhaustion of the resource will occur at the point where MEC+MUC=“reservation price” or “choke price”, if such a price exists.

• What does the choke price or reservation price represent?
Transition to a Backstop (substitute) Technology

Time at which non-renewable resource is exhausted and a backstop is discovered, thus shifting to the backstop use so that price of the non-ren. res. tapers off
What sets P*?
A Transition to Other Non-Renewables

• We can consider either:
  – Same resource, but ores of different quality (coal with high or low energy content); or
  – Different resources entirely (coal vs. oil)

• Multiple transitions, based on incremental exhaustion of “better” resources (less costly).

• Can think of backstop technology at end of process, as well.
Multiple non-renewable transitions, with backstop technology

Time

$\bar{p}$

$p_1$

$p_0$

$0$

MEC of backstop

Time at which non-renewable resource 1 is exhausted

Non-renewable resource 2 is exhausted

Non-renewable resource 3 is exhausted
Extraction Today Affects Future Costs: Increasing Marginal Extraction Costs

• Cost of extracting one unit of the resource increases as the stock gets smaller.

• Common terminology: “Stock effect”

• Now there is an extra cost to extracting today – the effect on future extraction costs.

• Disincentive to extract – extraction rate slows.

• Typically, with stock effects, exhaustion is not dynamically efficient.
Stock of a Non-renewable Resource

• How would we define the stock of a non-renewable resource?
  – Ore/reserves that are feasible to extract at current prices and technologies.

• Is this an exogenous entity?
  – How would technological change affect stock?
  – How does this create incentives for firms?
  – What about the choke price?
Will the Market Achieve Dynamic Efficiency?

• Yes, under certain assumptions, many of which are met in the markets for non-renewables.

• Private owners of resources will consider scarcity, not simply their extraction costs, or they risk missing out on a capital gain.

• Can we tell from market data whether the markets for non-renewables are dynamically efficient?
Conditions Under Which Dynamically Efficient Extraction Will Not Occur in Private Markets

• Non-competitive market structure (monopolies, cartels)

• Asymmetric information

• Incomplete markets
  – Externalities in production or consumption
  – Public goods
  – Tragedy of the commons/open access resources

• Divergence between private and social discount rates
Non-competitive Markets: Monopoly

- For monopolist, Hotelling Rule is slightly different:

\[ \frac{\Delta (MR - MEC)}{(MR - MEC)} = r \]

- For most “reasonable” demand functions, monopolist extracts more slowly, exhausts resource later than competitive private owner.

- Monopolist increases total profits from resource by restricting output in early time periods – monopoly rents.

- This is because restricting output raises the price in the early time periods and more profits can be reaped early rather than later, therefore slower extraction – total PV increases by restricting output in the early time periods.
For a monopolist, MC (S) is rising and not constant – increased supply only at higher price and thus control over supply of resource

At \( q_1 \rightarrow NB = P - C = p_1c_1 \)

which is greater than,

At \( q_2 \rightarrow NB = P - C = p_2c_2 \)
For a given price, $q_1 < q_2$ to equate $PV(MNB)_1 = PV(MNB)_2$

$PV(MB - MC)_1 = PV(MB - MC)_2$

$8 - 0.4 * q_1 - 2 = \frac{8 - 0.4 * q_2 - 2}{(1.10)^1}$

$q_1 + q_2 = 20$ so $q_2 = 20 - q_1$

Substituting:

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$\Rightarrow q_1^* = 10.239, \quad q_2^* = 20 - q_1^* = 9.761$

$p_1 = 8 - (0.4 * 10.239) = $3.90

$p_2 = 8 - (0.4 * 9.761) = $4.10

For a given price, $q_1 < q_2$ to equate $PV(MB1) = PV(MB2)$
Exploration and Technological Progress

• Technological progress can shift the MEC function downward over time.

• Exploration and discovery can also shift the MEC function downward over time.

• Both technological R&D and exploration exhibit “diminishing returns” over time.

• While costs may fall initially, when diminishing returns set in, costs will begin to rise.
solve

• Demand = MB = 25 – 0.8q
• Supply = MC = 5/unit
• Stock = 40 units
• Discount rate, r = 10% = 0.10
• MUC = marginal user cost = P – MC
• t = time period

1. Find $q_1$ & $q_2$
2. Find $p_1$ & $p_2$
3. Find MUC in $t_1$ & $t_2$
Dynamically Efficient Allocation in the Two-period Model

\[ q_1 \rightarrow 0 \rightarrow q_2 \]
Dynamic Efficiency with Constant Marginal Extraction Costs

**Period #1**

P ($)

\[ p_1 = 8.81 \]

\[ MUC_1 = 3.81 \]

\[ q_1 = 20.24 \]

**Period #2**

P ($)

\[ p_2 = 9.19 \]

\[ MUC_2 = 4.19 \]

\[ q_2 = 19.76 \]

MUC is *marginal user cost*

MEC is *marginal extraction cost*
Conclusions/observations

• Because $q_1 > q_2$, $p_1 < p_2$
• Given MEC, higher $p_2$ implies higher MUC in future time periods
• For a given $p$, $q_1 < q_2$ to equate $PVMB_1 = PVMB_2$ $\Rightarrow$ $p > MC =$ monopoly rents
• As extraction continues, stock depletes so that MUC continuously increases for all future time periods